Lab Manual

Subject: Lab Course on Numerical Methods

For

Under Graduate Students

BY

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Preface

This laboratory manual on Numerical Methods is designed specifically for under graduate students, aligning with the curriculum prescribed by most undergraduate science programs. It aims to provide students with hands-on experience in implementing numerical techniques that form the foundation for solving mathematical problems computationally.Each practical in this manual is presented with the corresponding theory and methods which require to solve the practice examples.I hope this manual serves as a valuable resource for both students and instructors. Taking these sample questions instructors can design the different examples. Students can write the program in "C' language, Scilab, Python for these methods for effective working of all the methods and formulae. Constructive feedback and suggestions for improvement are always welcome.

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Practical 1: Errors
Theory

Absolute, relative and Percentage Errors:

1) Absolute Error:

$$\begin{aligned}\text{Error} &= \text{True or actual value} X - \text{Approximate value}(X_1)\\ \text{Absolute Error}(E_A) &= |\text{True or actual value} - \text{Approximate value}|\\ &= |X - X_1|\end{aligned}$$

2) Relative Error:

$$\text{Relative Error} = E_R = \frac{\text{Absolute Error}}{\text{True value}}$$

$$\text{Relative Error} = E_R = \frac{E_A}{X} = \frac{|X - X_1|}{X}$$

3) Percentage Error:

$$\text{Percentage Error} = E_P = 100E_R$$

$$\text{Percentage Error} = E_P = 100\left(\frac{E_A}{X}\right)$$

4) Rounding-off numbers:

The numbers given below are rounded-off to fout significant figures

2.57434023	to	2.574
8.6535869579223	to	8.654
46.06879334	to	46.07
0.365120653	to	0.3651
0.79627422	to	0.7963
0.0067433	to	0.006743

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Practical 1: Errors

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1. Round off the following numbers correct to 4 significant figurs.
   (a) 20.5763903
   (b) 0.004624872
   (c) 5738.8529344
   (d) 0.359439
   (e) 5.76639
2. Find the relative error of the number $\frac{5}{7}$ whose approximate value is 0.714.
3. Find the approximate value of π is 3.1427812 and its value is 3.14159265. Find the percentage error.
4. Round off the numbers 856.53678121260 and 0.01352874 to 4 significant figures and calculate the relative error.
5. The approximate value of the number are $\frac{1}{6}$ are given as 0.165, 0.166, 0.167. Find the best approximation for $\frac{1}{6}$.
6. An approximate value of e is 2.7195518 and its true value is given by $x = 2.71821828$. Find the relative error and percentage error.

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Practical 2: Bisection and Regula Falsi Method
Theory

Bisection Method:

Step 1: For given equation $f(x) = 0$, identify the interval (a, b) such that $f(a)f(b) < 0$ i.e $f(a)$ and $f(b)$ have opposite signs in which the root of equation lies.

Step 2: We take $x_0 = \dfrac{a+b}{2}$ and find $f(x_0)$. If $f(x_0) = 0$ the x_0 is the required root of equation.

Step 3: If $f(x_0) \neq 0$ the root is either between a and x_0 or x_0 and b. Identify the next interval by verifying whether $f(a)$ and $f(x_0)$ have opposite signs or $f(x_0)$ and $f(b)$ have opposite signs.

Step 4: Rename the interval as (a_1, b_1).
Take $x_1 = \dfrac{a_1 + b_1}{2}$ and find $f(x_1)$. If $f(x_1) = 0$ the x_1 is the required root of equation.

Step 5: If $f(x_1) \neq 0$, then repeat Step 3 and 4. Each such repetition or a round of the process is called an iteration. The process may continue to many iterations. We continue the iterations until value of x_i is nearly same as in last two iterations.

Regula Falsi Method:

Step 1: For given equation $f(x) = 0$, identify the interval (a, b) such that $f(a)f(b) < 0$ i.e $f(a)$ and $f(b)$ have opposite signs in which the root of equation lies.

Step 2: We take $x_0 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$ and find $f(x_0)$. If $f(x_0) = 0$ the x_0 is the required root of equation.

Step 3: If $f(x_0) \neq 0$ the root is either between a and x_0 or x_0 and b. Identify the next interval by verifying whether $f(a)$ and $f(x_0)$ have opposite signs or $f(x_0)$ and $f(b)$ have opposite signs.

Step 4: Rename the interval as (a_1, b_1).
Take $x_1 = \dfrac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)}$ and find $f(x_1)$. If $f(x_1) = 0$ the x_1 is the required root of equation.

Step 5: If $f(x_1) \neq 0$, then repeat Step 3 and 4. Each such repetition or a round of the process is called an iteration. The process may continue to many iterations. We continue the iterations until value of x_i is nearly same in last two iterations.

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Practical 2: Bisection and Regula Falsi Method

1. Find an approximate root of equation $x^3-2x-5=0$ using bisection method.
2. Find an approximate root of equation $xe^x = 2$ between 0 and 1 by Regula Falsi method.
3. Find a root of equation $x^3 - 9x + 1 = 0$ by bisection method. (5 iterations).
4. Use Regula- Falsi Method to find real root of equation $e^x - 4x = 0$.
(4 iterations)
5. Find the approximate root of the equation $x^3 - x - 4 = 0$ by Regula Falsi method.

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Practical 3: Newton Raphson method
Theory

Newton Raphson Method:

Step 1: For given equation $f(x) = 0$, identify the interval (a, b) such that $f(a)f(b) < 0$ i.e $f(a)$ and $f(b)$ have opposite signs in which the root of equation lies.

Step 2: Find the derivative function $f'(x)$.

Step 3: We take $x_0 = \dfrac{a+b}{2}$ as the initial root or any value from the interval (a, b) that is near to the exact root.

Step 4: The iterative formula to find the root of equation is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0, 1, 2, \cdots, n$$

Step 5: At each step check value of $f(x_i)$. If $f(x_i) = 0$, then x_i is the approximate root of the given equation.

Step 6: If $f(x_i) \neq 0$, then go to the next root using Step 4. We continue the iterations till value of x_i is nearly same in last two iterations or close to exact root.

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Practical 3: Newton Raphson Method

1. Find the approximate root of following equations using Newton Raphson Method.
   (a) $x^2 + 5x + 1 = 0$ in $(-1, 0)$
   (b) $e^x \cos x = 1.4$
   (c) $xe^x = 1$ in $(0, 1)$
   (d) $x^3 - 5x + 3 = 0$.
   (e) $4e^{-x} \sin x - 1 = 0$
   (f) $x^4 + x^2 - 80 = 0$
2. Find r^{th} root of the following.
   (a) $\sqrt[3]{37}$
   (b) $\sqrt{15}$
   (c) $\sqrt[5]{25}$

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Practical 4: Iterative Method
Theory

Iteration Method:

Step 1: For given equation $f(x) = 0$, identify the interval (a, b) such that $f(a)f(b) < 0$ i.e $f(a)$ and $f(b)$ have opposite signs in which the root of equation lies.

Step 2: Transform the equation $f(x) = 0$ in the form $x = \phi(x)$ such that $|\phi'(x) < 1|$ in an immediate neighbourhood of the root.

Step 3: We take $x_0 = \dfrac{a+b}{2}$ as the initial root or any value from the interval (a, b) which is near to the exact root.

Step 4: The first approximation is as:

$$x_1 = \phi(x_0)$$

Step 5: Likewise a still better approximation is computed by succesive substitutions i.e.

$$x_{i+1} = \phi(x_i), i = 1, 2, \cdots, n$$

If this sequence of approximate values $x_1, x_2, x_3, \cdots, x_{n+1}$ converges to a limit 'c', then 'c' is taken as the root of given equation.

Step 6: The rate of convergence of the iterative method is linear. This rate of convergence can be accelerated by using Aitken's Δ^2- method.

Step 7: The formula to obtain value of 'c' by Aitken's Δ^2- method is

$$c = x_{i+1} - \frac{(\Delta x_i)}{(\Delta^2 x_{i-1})}$$

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Practical 4: Iterative method

Ques: Find the approximate root of following equations using iterative Method. Apply Aitken's $-\Delta^2$ formula.

1. $x + \log x - 2 = 0$ in $(1, 2)$.
2. $2x = cosx + 3$ in $(1, 2)$.
3. $xe^x = 1$ in $(0, 1)$.
4. $x^2 - 2x - 3 = 0$ in $(3, 4)$.

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Practical 5: Difference Table

Theory

Differences

The forward and backward differences are calculated at equally spaced values of x.
Here,

$$\begin{aligned} y_0 &= f(x_0) \\ y_1 &= f(x_1) = f(x_0+h) \\ y_2 &= f(x_2) = f(x_0+2h) \\ y_3 &= f(x_3) = f(x_0+3h) \\ &\vdots \end{aligned}$$

In general,

$$y_n = f(x_n) = f(x_0+nh)$$

Forward Difference:
The first forward difference is defined by

$$\Delta f(x) = f(x+h) - f(x)$$

The symbol Δ denotes forward difference operator.
The **first order forward differences** are given as

$$\begin{aligned} \Delta y_0 &= y_1 - y_0 \\ \Delta y_1 &= y_2 - y_1 \\ \Delta y_2 &= y_3 - y_2 \\ &\vdots \\ \Delta y_{n-1} &= y_n - y_{n-1} \end{aligned}$$

The **second order forward difference** is defined by

$$\Delta^2 f(x) = \Delta f(x+h) - \Delta f(x)$$

The successive second order forward difference are given as

$$\begin{aligned} \Delta^2 y_0 &= \Delta y_1 - \Delta y_0 \\ \Delta^2 y_1 &= \Delta y_2 - \Delta y_1 \\ \Delta^2 y_2 &= \Delta y_3 - \Delta y_2 \\ &\vdots \\ \Delta^2 y_{n-2} &= \Delta y_{n-1} - \Delta y_{n-2} \end{aligned}$$

Similarly, third order forward differences are given as

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$
$$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$$
$$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$$
$$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$$

and so on......

The forward differences are shown in the following table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\Delta y_0 = y_1 - y_0$			
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		
		$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

Backward Difference:
The first backward difference is defined by

$$\nabla f(x) = f(x) - f(x-h)$$

The symbol ∇ denotes backward difference operator.

Thus in particular, the first order backward differences are given as

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\nabla y_3 = y_3 - y_2$$
$$\vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

The second order backward difference are defined as

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$$
$$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$$
$$\vdots$$
$$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$$

and so on....

The backward differences are shown in the following table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		$\nabla y_1 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$	
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	
x_3	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$		
		$\nabla y_4 = y_4 - y_3$			
x_4	y_4				

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Practical 5: Difference Table

1. Construct the difference table for the follwing data and obtain the value of 1) Δy_1 2) $\Delta^2 y_2$ 3) $\Delta^5 y_0$ 4) $\Delta^3 y_4$ 5) ∇y_2 6) $\nabla^2 y_5$ 7) $\nabla^5 y_5$ 8) $\nabla^4 y_3$ 9) $\nabla^3 y_2$

x	0	5	10	15	20	25
y	6	10	13	17	23	31

2. Construct the difference table for the following data and obtain the value of 1) ∇y_2 2) $\nabla^2 y_5$ 3) $\nabla^5 y_5$ 4) $\nabla^4 y_3$ 5) $\nabla^3 y_2$

Year	1911	1921	1931	1941	1951	1961
Population	12	15	20	27	39	52

3. Construct the difference table for the following data. Write the forward and backward difference at each stage.
1)

Year	-1	0	1	2	3	4	5
Population	-13	-7	-1	11	35	77	143

2)

x	$1 \cdot 00$	$1 \cdot 25$	$1 \cdot 50$	$1 \cdot 75$	$2 \cdot 00$
y	$0 \cdot 3678$	$0 \cdot 2865$	$0 \cdot 2231$	$0 \cdot 1737$	$0 \cdot 1353$

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Practical 6: Newton's Forward Interplation
Theory

The Newton's Gregory formula for Forward Interpolation

Step 1: For given data, construct the forward difference table.

Step 2: Apply the Newton's Gregory formula for Forward Interpolation given by

$$y = f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots\cdots\cdots + \frac{u(u-1)(u-2)\cdots(u-(n-1))}{n!}\Delta^n y_0$$

$$\text{where} \quad u = \frac{x - x_0}{h}$$

Shift Operator: The First order Shift Operator is defined as

$$\text{Shift Operator} = Ef(x) = f(x+h)$$
$$Ey_0 = y_1$$
$$Ey_1 = y_2$$
$$\vdots$$

The Second order Shift Operator is defined as

$$E^2 y_0 = y_2$$
$$E^2 y_1 = y_3$$
$$\vdots$$

In general

$$E^n y_k = y_{k+n}$$

The relation between forward difference operator and shift operator is given as

$$\Delta \equiv E - 1$$
$$\Delta^n \equiv (E-1)^n$$

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Practical 6: Newton's Forward Interpolation

1. From the following data, find y when $x = 1 \cdot 42$

x	1	$1 \cdot 2$	$1 \cdot 4$	$1 \cdot 6$	$1 \cdot 8$	2
y	$0 \cdot 0$	$-0 \cdot 112$	$-0 \cdot 016$	$0 \cdot 336$	$0 \cdot 992$	2

2. Estimate the value $x = e^{0 \cdot 24}$ using the following data.

x	$0 \cdot 1$	$0 \cdot 2$	$0 \cdot 3$	$0 \cdot 4$	$0 \cdot 5$
e^x	$1 \cdot 10517$	$1 \cdot 22140$	$1 \cdot 34866$	$1 \cdot 49182$	$1 \cdot 64872$

3. Estimate the missing term in the following data:

x	0.1	0.2	0.3	0.4	0.5
y	1.4	?	1.76	2.00	2.28

4. Find the interpolation polynomial which corresponds to the following data.

x	0	1	2
y	2	7	12

5. Obtain value of y at $x = 1.97$ using the following data.

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

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Practical 7: Newton's Backward Interpolation
Theory

The Newton's Gregory formula for Backward Interpolation

Step 1: For given data, construct the backward difference table.

Step 2: Apply the Newton's Gregory formula for Backward Interpolation is

$$y = f(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots\cdots\cdots + \frac{u(u+1)(u+2)\cdots(u+(n-1))}{n!}\nabla^n y_n$$

$$\text{where} u = \frac{x - x_n}{h}$$

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Practical 7: Newton's Backward Interpolation

1. Find value of y at $x = 3.56$ using the following data.

x	$1\cdot5$	$2\cdot0$	$2\cdot5$	$3\cdot0$	$3\cdot5$	4
y	$33\cdot75$	$7\cdot00$	$13\cdot625$	$24\cdot00$	$38\cdot875$	$59\cdot00$

2. Obtain value of y at $x = 2.03$ using the following data.

x	1.96	1.98	2.00	2.02	2.04
y	0.7825	0.7739	0.7651	0.7563	0.7473

3. Find value of tan(0.27) using Newton's backward interpolation formula.

x	$o\cdot20$	$0\cdot22$	$0\cdot24$	$0\cdot26$	$0\cdot28$	$0\cdot30$
$\tan x$	$0\cdot2027$	$0\cdot2236$	$0\cdot2447$	$0\cdot2660$	$0\cdot2875$	$0\cdot3093$

4. Compute $f(2.45)$ from the following table:

x	$0\cdot00$	$0\cdot50$	$1\cdot00$	$1\cdot50$	$2\cdot00$	$2\cdot50$	$3\cdot00$
$f(x)$	$0\cdot000$	$0\cdot191$	$0\cdot341$	$0\cdot433$	$0\cdot477$	$0\cdot494$	$0\cdot499$

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Practical 8: Lagrange's Interpolation Formula
Theory

The Lagrange's Interpolation formula is given by

$$\begin{aligned} y = f(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)\cdots\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\cdots\cdots(x_0-x_n)}y_0 \\ &+ \frac{(x-x_0)(x-x_2)(x-x_3)\cdots\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots\cdots(x_1-x_n)}y_1 \\ &+ \frac{(x-x_0)(x-x_1)(x-x_3)\cdots\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\cdots\cdots(x_1-x_n)}y_2 \\ &\vdots \\ &\frac{(x-x_0)(x-x_1)(x-x_2)\cdots\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\cdots\cdots(x_0-x_{n-1})}y_n \end{aligned}$$

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Practical 8: Lagrange's Interpolation Formula

1. Find value of y at $x = 9$ using Lagrange's interpolation formula, given that:

x	1	3	4	8	10
y	8	15	19	32	40

2. Find the cubic polynomial which takes the following values.

x	0	1	2	3
y	1	0	1	10

3. Find value of $y(2)$ using the Lagrange's interpolation formula using the data.

x	0	1	3	4
y	-12	0	1 2	24

4. Find the form of polynomial from the following data.

x	0	1	2	5
y	2	3	12	147

5. Find value of x such that $f(x) = 1.308656$ using the Lagrange's interpolation formula using following data.

x	3	3.5	4
f(x)	1.09861	1.25277	1.3863

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Practical 9: Newton's Divided Difference Formula
Theory

Newton's Divided Difference Formula

The Newton's divided difference table:

x	y	I^{st} Div. Diff.	II^{nd}Div. Diff.	III^{rd} Div. Diff.	IV^{th} Div. Diff.
x_0	y_0				
		$[x_0, x_1] = \dfrac{y_1 - y_0}{x_1 - x_0}$			
x_1	y_1		$[x_0, x_1, x_2] = \dfrac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0}$		
		$[x_1, x_2] = \dfrac{y_2 - y_1}{x_2 - x_1}$		$[x_0, x_1, x_2, x_3] = \dfrac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{x_3 - x_0}$	
x_2	y_2		$[x_1, x_2, x_3] = \dfrac{[x_2, x_3] - [x_1, x_2]}{x_3 - x_1}$		$[x_0, x_1, x_2, x_3, x_4] = \dfrac{[x_1, x_2, x_3, x_4] - [x_0, x_1, x_2, x_3]}{x_4 - x_0}$
		$[x_2, x_3] = \dfrac{y_3 - y_2}{x_3 - x_2}$		$[x_1, x_2, x_3, x_4] = \dfrac{[x_2, x_3, x_4] - [x_1, x_2, x_3]}{x_4 - x_1}$	
x_3	y_3		$[x_2, x_3, x_4] = \dfrac{[x_3, x_4] - [x_2, x_3]}{x_4 - x_2}$		
		$[x_3, x_4] = \dfrac{y_4 - y_3}{x_4 - x_3}$			
x_4	y_4				

The Newton's divided difference formula is given as

$$
\begin{aligned}
y(x) = y_0 &+ (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] \\
&+ (x - x_0)(x - x_1)(x - x_2)[x_0, x_1, x_2, x_3] + \cdots\cdots\cdots\cdots
\end{aligned}
$$

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Department of Mathematics
S.Y.B.Sc.(Mathematics)
Subject: Lab Course on Numerical Methods
Practical 9: Newton's Divided Difference Formula

1. Prepare Newton's divided difference table for the following values of x and y.

x	1	2	3	6	8	9
y	4	6	9	12	18	20

2. Find $f(3)$ using Newton's divided difference formula, given that:

x	1	3	4	8	10
y	8	15	19	32	40

3. Find the cubic polynomial which takes the following values using Newton's divided difference formula.

x	0	1	2	3
y	1	0	1	10

4. Find value of $y(2)$ using Newton's divided difference formula from the following data.

x	0	1	3	4
y	-12	0	12	24

5. Find the form of function using Newton's divided difference formula from the following data.

x	3	2	1	-1
y	-1	8	11	-25

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Subject: Lab Course on Numerical Methods
Practical 10: Numerical Differentiation
Theory

Derivatives using Newton's Forward Difference Formula:

Newton's formula for Forward Interpolation is given by

$$\begin{aligned} y = f(x) &= y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 \\ &\quad + \cdots\cdots\cdots + \frac{u(u-1)(u-2)\cdots(u-(n-1))}{n!}\Delta^n y_0 \\ &\quad \text{where } \; u = \frac{x-x_0}{h} \end{aligned} \tag{1}$$

Differentiating equation (1) w.r.t. x, we get the formula for first order derivative as,

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \cdots\cdots\right] \tag{2}$$

If $x = x_0$, then the formula for first order derivative at $x = x_0$ is given as

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\cdots\right]$$

If $x = x_1$, then the formula for first order derivative at $x = x_1$ is given as

$$\left(\frac{dy}{dx}\right)_{x=x_1} = \frac{1}{h}\left[\Delta y_1 - \frac{1}{2}\Delta^2 y_1 + \frac{1}{3}\Delta^3 y_1 - \frac{1}{4}\Delta^4 y_1 + \cdots\cdots\right]$$

Differentiating the equation (2) w.r.t. x, we get the formula for second order derivative as,

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{(6u^2-18u+11)}{12}\Delta^4 y_0 + \cdots\cdots\right]$$

If $x = x_0$, then the formula for second order derivative at $x = x_0$ is given as

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\cdots\right]$$

Note: The formula for first order and second order can also be obtained using Newton's formula for Backward Interpolation in a similar manner.

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Department of Mathematics
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Subject: Lab Course on Numerical Methods
Practical 10: Numerical Differentiation

1. The following table gives the angular displacements θ (radians) at different intervals of time t (seconds)

θ	$0 \cdot 052$	$0 \cdot 105$	$0 \cdot 168$	$0 \cdot 242$	$0 \cdot 327$	$0 \cdot 408$	$0 \cdot 489$
t	0	$0 \cdot 02$	$0 \cdot 04$	$0 \cdot 06$	$0 \cdot 08$	$0 \cdot 10$	$0 \cdot 12$

Calculate the angular velocity at the instant $t = 0 \cdot 052$

2. Compute $f'(1 \cdot 12)$ and $f''(1 \cdot 12)$ from the following table

x	$1 \cdot 11$	$1 \cdot 12$	$1 \cdot 13$	$1 \cdot 14$	$1 \cdot 15$	$1 \cdot 16$
$f(x)$	$6 \cdot 2321$	$6 \cdot 2544$	$6 \cdot 2769$	$6 \cdot 2996$	$6 \cdot 3225$	$6 \cdot 3456$

3. A slider in a machine moves along a fixed straight rod. it's distance 'x' centimeter along the rod is given below for various values of time t (seconds). Find the velocity and acceleration of the slider at $t = 0.2$.

Time t	$0 \cdot 0$	$0 \cdot 1$	$0 \cdot 2$	$0 \cdot 3$	$0 \cdot 4$	$0 \cdot 5$	$0 \cdot 6$
Displacement$y$	$30 \cdot 13$	$31 \cdot 62$	$32 \cdot 87$	$33 \cdot 64$	$33 \cdot 95$	$33 \cdot 81$	$33 \cdot 24$

4. Obtain value of $\frac{dy}{dx}$ at $x = 7.48$.

x	$7 \cdot 47$	$7 \cdot 48$	$7 \cdot 49$	$7 \cdot 50$	$7 \cdot 51$	$7 \cdot 52$
y	$1 \cdot 92$	$1 \cdot 95$	$1 \cdot 98$	$2 \cdot 01$	$2 \cdot 03$	$2 \cdot 06$

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S.Y.B.Sc.(Mathematics)
Subject: Lab Course on Numerical Methods
Practical 11: Numerical Integration
Theory

A General Quadrature Formula:

$$I = \int_{a=x_0}^{b=x_0+nh} y dx = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \left(\frac{n^3}{3} - \frac{n^2}{2}\right)\frac{\Delta^2 y_0}{2!} + \left(\frac{n^4}{4} - n^3 + n^2\right)\frac{\Delta^2 y_0}{2!} + \cdots\cdots\right]$$

B Trapezoidal Rule:

$$I = \int_a^b y dx = \frac{h}{2}\left[(y_0 + y_n) + 2\left(y_1 + y_2 + y_3 + \cdots + y_{n-1}\right)\right]$$

C Simpson's One Third Rule:

$$I = \int_a^b y dx = \frac{h}{3}\left[(y_0 + y_n) + 2\left(y_2 + y_4 + \cdots + y_{n-2}\right) + 4\left(y_1 + y_3 + \cdots + y_{n-1}\right)\right]$$

D Simpson's Three Eighth Rule:

$$I = \int_a^b y dx = \frac{3h}{8}\left[(y_0 + y_n) + 2\left(y_3 + y_6 + \cdots + y_{n-3}\right) + 3\left(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}\right)\right]$$

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Practical 11: Numerical Integration

1. Evaluate $\int_0^6 e^x\, dx$ using *Simpson's One-Third Rule.*

2. Evaluate $\int_0^{\pi/4} \tan x\, dx$ using *Trapezoidal Rule.*

3. Find value of $\int_1^{2\cdot 2} \ln x\, dx$ by *Simpson's Three-Eighth Rule*

4. Evaluate $\int_2^9 \frac{1}{1+x^2}\, dx$ using *Trapezoidal Rule.*

5. The velocities of a car (running on single straight road) at intervals of 2 minutes are given below:

Time in minutes	0	2	4	6	8	10	12
Velocity in km/hr.	0	22	30	27	18	7	0

Apply *Simpson's Rule* to find the distance covered by the car.

6. Calculate the area bounded by the curve using the following data:

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	23	19	14	11	12.5	16	19	20	20

7. A solid of revolution is formed by rotating about the X-axis , the area between the X-axis, the lines x=0 and x=1 and a curve through the points with the following co-ordinates:

x	0.00	0.25	0.50	0.75	1
y	1.00	0.9896	0.5589	0.9089	0.8415

Compute the volume of the solid formed. (The volume of the solid = $\pi \int_a^b y^2 dx$)

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Department of Mathematics

S.Y.B.Sc.(Mathematics)

Subject: Lab Course on Numerical Methods

Practical 12: Euler's Modified method

Theory

Given Differential Equation:

$\frac{dy}{dx} = f(x, y)$ with initial conditions at $x = x_0,\ y = y_0$ i.e. $y(x_0) = y_0$

We have to find, the next value of y at given value of x. i.e. find y_1 at given value x_1. The value x_1 is estimated as $x_1 = x_0 + h$ where h is the step size.

1) Euler's Method:

The solution of differential equation $\frac{dy}{dx} = f(x, y)$ with initial conditions at $x = x_0,\ y = y_0$ for $x = x_1, x_2, \cdots, x_n$ is obtained by using the following formula

$$y_{n+1} = y_n + hf(x_n, y_n), \qquad n = 0, 1, 2, \cdots\cdots$$
$$\text{where } x_n = x_0 + kh, \qquad k = 0, 1, 2, \cdots\cdots$$

2) Euler's Modified Method:

a) **Initial Step:**
To start, we use Euler's formula to find value of y at $x = x_1$. Compute the initial value of y_1 as

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

b) **Iterative Step:**
Use $y_1^{(0)}$ as initial value for first iteration to find value of y_1. To find more correct approximation at $x = x_1$, the Euler's modified formula is given as

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right], \qquad n = 0, 1, 2, \cdots\cdots$$

c) To find y_2, the initial value is calculated as

$$y_2^{(0)} = y_1 + hf(x_1, y_1)$$

d) **Iterative Step:**
Use $y_2^{(0)}$ as initial value for first iteration to find value of y_2. The iterative formula is given as

$$y_2^{(n+1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(n)})\right], \qquad n = 0, 1, 2, \cdots\cdots$$

e) Continue as above to calculate values of $y_3, y_4, y_5, \cdots\cdots$

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Department of Mathematics
S.Y.B.Sc.(Mathematics)
Subject: Lab Course on Numerical Methods
Practical 12: Euler's Modified method

1. Solve by Euler's method: the equation $\frac{dy}{dx} = xy$ with $y(0) = 1$ and find $y(0.05)$ by taking $h = 0.01$.
2. Find value of $y(0.6)$ using Euler's modified method. Given differential equation $\frac{dy}{dx} = x + y$ with initial condition $y = 0$ when $x = 0$. Take $h = 0.3$.
3. Given that $\frac{dy}{dx} = 1 + y^2$ with $y(1) = 1$. Find $y(1.5)$ by Euler's method. Take $h = 0.1$.
4. Given $\frac{dy}{dx} = x^2 + y$ with $y(0) = 2$. Obtain $y(0.4)$ using Euler's Modified method.
5. Solve the differential equation $\frac{dy}{dx} = x + 2y^3$ by Euler's method with the condition $y(0) = 1$. Take $h = 0.2$ and obtain $y(0.2)$, $y(0.4)$, $y(0.6)$, $y(0.8)$ and $y(1)$.

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Department of Mathematics
S.Y.B.Sc.(Mathematics)
Subject: Lab Course on Numerical Methods
Practical 13: Runge Kutta method
Theory

Given Differential Equation:

$\frac{dy}{dx} = f(x,y)$ with initial conditions at $x = x_0,\ y = y_0$ i.e. $y(x_0) = y_0$

1) Runge Kutta Second order Method:

$$\begin{aligned} y_1 &= y_0 + \frac{1}{2}(k_1 + k_2) \\ \text{where } k_1 &= hf(x_0, y_0) \\ k_2 &= hf(x_0 + h, y_0 + k_1) \\ \text{Similarly, } y_2 &= y_1 + \frac{1}{2}(k_1 + k_2) \\ \text{where } k_1 &= hf(x_1, y_1) \\ k_2 &= hf(x_1 + h, y_1 + k_1) \end{aligned}$$

and so on for next values of y.

1) Runge Kutta Fourth order Method:

$$\begin{aligned} y_1 &= y_0 + \frac{1}{6}(k_1 + k_2 + k_3 + k_4) \\ \text{where } k_1 &= hf(x_0, y_0) \\ k_2 &= hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) \\ k_3 &= hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) \\ k_4 &= hf(x_0 + h, y_0 + k_3) \\ \text{Similarly, } y_2 &= y_1 + \frac{1}{6}(k_1 + k_2 + k_3 + k_4) \\ \text{where } k_1 &= hf(x_1, y_1) \\ k_2 &= hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}) \\ k_3 &= hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}) \\ k_4 &= hf(x_1 + h, y_1 + k_3) \end{aligned}$$

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Department of Mathematics

S.Y.B.Sc.(Mathematics)

Subject: Lab Course on Numerical Methods

Practical 13: Runge Kutta method

1. Determine $y(3)$ using second order Runge-Kutta formula. Given that $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(2) = 1$. Take $h = 0.5$.
2. Obtain value of $y(1.3),\ y(1.6)$ using Runge Kutta second order formula for differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(1) = 4$. Take $h = 0.3$.
3. Use Runge Kutta fourth order formula to find value of $y(0.5)$. Given that $\frac{dy}{dx} = xy^2$, with $y(0) = 2$. Take $h = 0.5$.
4. Generate $y(0.2),\ y(0.4)$ using fourth order Runge-Kutta formula for $\frac{dy}{dx} = -2y$ with $y(0) = 1$. (Take $h = 0.2$)
5. Given that $\frac{dy}{dx} = x + y$, with $y(0) = 5$. Find $y(0.2)$ using Runge Kutta second order formula. Take $h = 0.1$

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Subject: Lab Course on Numerical Methods

Practical 14: Curve fitting

Theory

A] Method of Least Squares:

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3), \cdots\cdots$ be the set of points given. Let $y = a + bx + cx^2 + \cdots\cdots + kx^n$ be the polynomial of degree n be fitted to this data. At x_i, $(i = 1, 2, \cdots\cdots, m)$, the given value is y_i and the corresponding value on fitting curve is $f(x_i)$. If e_i is the error of approximation at $x = x_i$, then we write

$$e_i = y_i - f(x_i)$$

For m points, we write

$$\begin{aligned} S &= [y_1 - f(x_1)]^2 + [y_2 - f(x_2)]^2 + \cdots\cdots + [y_m - f(x_m)]^2 \\ &= e_1^2 + e_2^2 + \cdots\cdots + e_m^2 \end{aligned}$$

We want to minimise S. The method of minimizing the sum of the squares of error i.e. minimising S is known as the method of least squares.

A] Fitting a Straight Line:

Let $y = a + bx$ be the straight line to be fitted to the given data $(x_1, y_1), (x_2, y_2), (x_3, y_3), \cdots\cdots$, ($x_m$
The normal equations to fit the linear equation are

$$\sum_{i=1}^{m} y_i = ma + b\sum_{i=1}^{m} x_i$$

$$\sum_{i=1}^{m} x_i y_i = a\sum_{i=1}^{m} x_i + b\sum_{i=1}^{m} x_i^2$$

B] Fitting a Second degree polynomial:

Let $y = a + bx + cx^2$ be the second degree polynomial to be fitted to the given data $(x_1, y_1), (x_2, y_2), (x_3, y_3), \cdots\cdots, (x_m, y_m)$
The normal equations to fit the linear equation are

$$\sum_{i=1}^{m} y_i = ma + b\sum_{i=1}^{m} x_i + c\sum_{i=1}^{m} x_i^2$$

$$\sum_{i=1}^{m} x_i y_i = a\sum_{i=1}^{m} x_i + b\sum_{i=1}^{m} x_i^2 + c\sum_{i=1}^{m} x_i^3$$

$$\sum_{i=1}^{m} x_i^2 y_i = a\sum_{i=1}^{m} x_i^2 + b\sum_{i=1}^{m} x_i^3 + c\sum_{i=1}^{m} x_i^4$$

* * *

C] Fitting a Power function:

Let $y = cx^d$ be the second degree polynomial to be fitted to the given data $(x_1, y_1), (x_2, y_2), (x_3, y_3), \cdots\cdots, (x_m, y_m)\dot{}$Consider a power function

$$y = cx^d$$

Taking logarithms on both sides, we get

$$\log y = \log c + d \log d$$

which is of the form

$$v = a + bu$$

The values of a and b can be obtained using normal equations for fitting the linear polynomial and then values of c and d.

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Department of Mathematics
S.Y.B.Sc.(Mathematics)
Subject: Lab Course on Numerical Methods
Practical 14: Curve fitting

1. The table gives the temperature T (in ^0C) and lengths l (in mm) of heated rod. If $l = a_0 + a_1 T$, find the values of a_0 and a_1 using least squares method:

T	40	50	60	70	80
l	600.5	600.6	600.8	600.9	601.0

2. Find the best values of a_0, a_1 and a_2 so that the parabola $y = a_0 + a_1 x + a_2 x^2$ fits the data:

x	0.78	1.56	2.34	3.12	3.81
y	2.50	1.20	1.12	2.25	4.28

3. Determine the constants a and b by the least squares method such that $y = ae^{bx}$ fits the following data:

x	1	2	3	4	5	6	7	8
y	15.3	20.5	27.4	36.6	49.1	65.6	87.8	117.6

4. Fit a function of the form $y = ax^b$ to the following data:

x	2	4	7	10	20	40	60	80
y	43	25	18	13	8	5	3	2

5. Fit a linear function for the following data:

x	6	8	10	12	14	16	18	20	22	24
y	3.8	3.7	4	3.9	4.3	4.2	4.2	4.4	4.5	4.5

6. Fit the curve $y = cx^d$ to the following data:

x	2.2	2.7	3.5	4.1
y	65	60	53	50

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Subject: Lab Course on Numerical Methods

Practical 15: Gauss Seidel iteration method

Theory

Consider the system of linear equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

We write these equations as

$$x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}}x_2 - \frac{a_{13}}{a_{11}}x_3 \quad (3)$$
$$x_2 = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}}x_1 - \frac{a_{23}}{a_{22}}x_3 \quad (4)$$
$$x_3 = \frac{b_3}{a_{33}} - \frac{a_{31}}{a_{33}}x_1 - \frac{a_{32}}{a_{33}}x_2 \quad (5)$$

We start with the initial values

$$x_2 = x_2^{(0)} = 0 \text{ and } x_3 = x_3^{(0)} = 0 \text{ to find } x_1^{(1)},\ x_2^{(1)},\ x_3^{(1)}$$

where $x_1^{(1)},\ x_2^{(1)},\ x_3^{(1)}$ are first approximations to $x_1,\ x_2,\ x_3$ respectively.
To find $x_1^{(1)}$, put $x_2 = x_2^{(0)} = 0$ and $x_3 = x_3^{(0)} = 0$ in equation (3).
To find $x_2^{(1)}$, put $x_1 = x_1^{(1)}$ and $x_3 = x_3^{(0)} = 0$ in equation (4).
To find $x_3^{(1)}$, put $x_1 = x_1^{(1)}$ and $x_2 = x_2^{(1)}$ in equation (5).

After first approximation, we get values of $x_1^{(1)},\ x_2^{(1)},\ x_3^{(1)}$
Next we want to find $x_1^{(2)},\ x_2^{(2)},\ x_3^{(2)}$
To find $x_1^{(2)}$, put $x_2 = x_2^{(1)}$ and $x_3 = x_3^{(1)}$ in equation (3).
To find $x_2^{(2)}$, put $x_1 = x_1^{(2)}$ and $x_3 = x_3^{(1)}$ in equation (4).
To find $x_3^{(2)}$, put $x_1 = x_1^{(2)}$ and $x_2 = x_2^{(2)}$ in equation (5).
Thus we get values of $x_1^{(2)},\ x_2^{(2)},\ x_3^{(2)}$ after second approximation to $x_1,\ x_2,\ x_3$.
This procedure is repeated till the values of $x_1,\ x_2$ and x_3 are obtained to desired accuracy.

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Subject: Lab Course on Numerical Methods

Practical 15: Gauss Seidel iteration method

Solve the following system of linear equations by Gauss Seidel method. (upto 4 iterations)

1.
$$\begin{aligned} 28x + 4y - z &= 32 \\ 2x + 17y + 4z &= 35 \\ x + 3y + 10z &= 24 \end{aligned}$$

2.
$$\begin{aligned} 2x_1 - x_2 + x_3 &= 5 \\ x_1 + 3x_2 - 2x_3 &= 7 \\ x_1 + 2x_2 + 3x_3 &= 10 \end{aligned}$$

3.
$$\begin{aligned} 3x + 8y + 29z &= 71 \\ 83x + 11y - 4z &= 95 \\ 7x + 52y + 13z &= 104 \end{aligned}$$

4.
$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ 2x_1 + 10x_2 + x_3 &= 13 \\ x_1 + x_2 + 5x_3 &= 7 \end{aligned}$$

5.
$$\begin{aligned} 6x + y + z &= 105 \\ 4x + 8y + 3z &= 155 \\ 5x + 4y - 10z &= 65 \end{aligned}$$

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